# Robust OFDM Timing Synchronisation

C. Williams, M.A. Beach
Centre for Communications Research
Bristol University, Woodland Road
Bristol, BS8 1UB, UK
{chris.williams, m.a.beach}@bristol.ac.uk

S. McLaughlin
Institute for Digital Communications
Edinburgh University, Mayfield Road
Edinburgh, EH9 3JL, UK
sml@ee.ed.ac.uk

***Abstract*—A new pre-FFT synchronization method for OFDM is proposed and assessed that gives improved performance in multipath channels. The technique can be used for a range of OFDM signal parameters, and channel environments. The relative increase in complexity over existing correlation based methods is less than 10%.**

## I. INTRODUCTION

This paper addresses pre-FFT synchronisation for orthogonal frequency division multiplex (OFDM) technologies. Such algorithms need to be robust to varying multipath conditions including transmissions from multiple transmitters as in a broadcast single frequency network (SFN). To ensure the most efficient data transmission possible, there should be no constraints on how much of the cyclic prefix (CP) is occupied by intersymbol interference (ISI). In the general case preamble symbols are not available and therefore the technique derived here does not assume the presence of such a preamble.

In particular a solution for timing synchronisation is proposed that is robust even when the strongest multipath components are delayed relative to the first arriving paths. In this situation existing methods perform poorly.

## II. OFDM SYNCHRONISATION APPROACHES

Synchronisation algorithms for OFDM can be divided into pre-FFT and post-FFT classes. The primary goal of pre-FFT processing is to provide a symbol of data to the FFT process, such that ISI and ICI are minimised otherwise the output from the FFT will be degraded. Any delay of the timing point will introduce ISI. To mitigate against intersymbol interference (ISI) a guard interval (or cyclic prefix, CP) of $N_g$ samples is inserted before each symbol of N samples. The periodic nature of the DFT is exploited by making the CP a replica of the last $N_g$ samples of the symbol. The CP is chosen to exceed the largest expected multipath delay.

For pre-FFT synchronisation the structure of the symbol needs to be exploited, either using the CP [1] or using short inserted repeating sequences [2]. However, since both methods produce synchronisation estimates using a correlation process, the same form of estimator can be used in both cases.

Exploiting the redundancy introduced by the CP to estimate time and frequency parameters is most commonly carried by averaging the correlation between the CP and the end of the useful symbol as analysed in [1]. For timing estimation in additive white Gaussian noise (AWGN) the maximum likelihood function consists of a summed correlation term and an energy correction term (*E*), as shown in (1) (see [1] for details). In this paper (1) will be called the correlation function.

$$\gamma(m) = \sum_{k=m}^{m+N_g-1} r(k)r^*(k+N) + E \qquad (1)$$

Where *r(k)* is the received signal. A two-step optimisation process first estimates the timing offset (assuming perfect frequency offset) based on the peak of (1), and then the frequency offset is calculated based on the phase shift between the CP and the end of the symbol. In dispersive environments the performance is degraded since the correlation will include ISI. This ISI corruption is more severe for short guard intervals, where the proportion of ISI free CP is limited.

In AWGN, the correlation function $\gamma(m)$ from (1) is a triangular function, the ideal timing point being at the peak, and the length of the slopes is the CP interval ($T_g$). Under some mild assumptions, it can be shown that in a multipath environment the output of the correlator is the summation of triangular functions, $\gamma_i(m)$, one for each multipath component delayed and weighted appropriately. Thus for P paths:

$$\gamma_P(m) = \sum_{i=1}^{P} \gamma_i(m) \qquad (2)$$

In multipath channels the peak of $\gamma_P(m)$ does not necessarily point to the position of the first arriving path, as demonstrated in Figure 1. In this situation ISI will occur due to the delayed timing estimate.

For dispersive channels, approaches that detect the leading edge of the correlator output may provide improved performance [3]. A straightforward way to do this is to set a threshold, and detect the crossing of this threshold. For complex channels and for varying SNR, the amplitude of the correlation characteristic will change in time, so the threshold needs to be set relative to this peak. However, performance is determined by the multipath characteristic.

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This influence can be reduced by fitting (least squares) a line to the leading edge, and applying the threshold to this line.

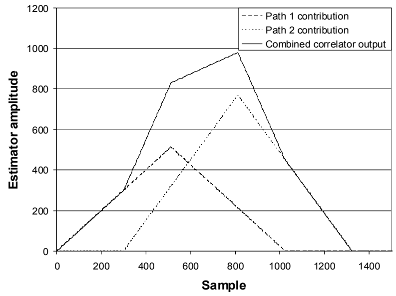


Figure 1. Summed correlation function for two path channel

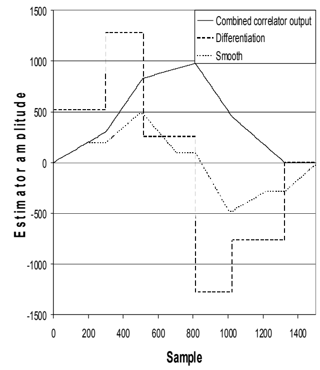


Figure 2. Derivative estimation method

## III. CORRELATION DERIVATIVE METHOD

Fig. 1 has shown that peak detection from the correlation function can give a high estimation error in multipath environments.

Each multipath component adds its own weighted and delayed triangular function $\gamma_i(m)$ (Fig. 1), and each of these functions rise over a period $T_g$ and then fall over a further period $T_g$. When the maximum multipath delay is less the $T_g$, for the period corresponding to the rising edge of the first multipath component (0 to 512 in the figure above), the triangular functions $\gamma_i(m)$ of the other components are being added in, and all are rising. After the peak of the first component, the function $\gamma_i(m)$ starts to fall, and the other functions will also in turn stop increasing and fall. Therefore up to the peak of the first component, the slope of the combined correlator output $\gamma_P(m)$ is monotonically increasing (no noise). After the peak position of the first component, the slope of $\gamma_P(m)$, though possibly positive, starts to decrease. Therefore the ideal timing point (when the multipath is bounded by $N_g$) is the point at which the derivative of the function $\gamma_P(m)$ starts to decrease, regardless of the channel power delay profile. The dashed line in Fig. 2 demonstrates this.

In practice noise will corrupt the estimation of the derivative, and a one point estimator (subtracting adjacent samples) is too noisy to be useful. A simple average of one point estimates gives the dotted line in Fig. 2. For different CP lengths, a good compromise for choice of this filter length was half of the CP length. The estimation problem now is to find the point at which this averaged signal starts to fall after its peak. In this work, the falling edge is projected backwards (LS fit) to find the timing point.

It is found that synchronisation parameter estimates have occasionally a large error, but these are isolated events. It is found that using a median filter is an effective method of removing these spurious results. To improve estimator performance, short filters have been added to the estimates. For $N_g$ of 64 samples ($N_u$ is 2048 samples) a 15 point median filter followed by a 16 point FIR filter have been used to good effect. For longer CPs, shorter filters can be used.

## IV. PERFORMANCE

For this investigation a DVB-T system [4] has been simulated, using the 2k-mode with 16QAM modulation, 64 point CP. To create a challenging multipath environment, a channel representative of a single frequency network (SFN) has been used, with two transmitters. The channel model for each transmitter is a single tap Ricean channel, with K-factor of -4.8dB and the deterministic component has a relative frequency offset of 0.33 compared to the maximum Doppler frequency [5]. The channel is thus parameterised as a function of the relative delay and power of the two components. Eb/No is 20dB in the results presented, but the algorithms are robust down to below 5dB. The benefits remain for other channels.

Figure 3 presents the estimator's timing error performance. The new derivative method shows a marked improvement in performance over the other techniques for ISI confined to the CP. With the estimate filtering proposed, the error variance for the correlation and derivative methods were similar

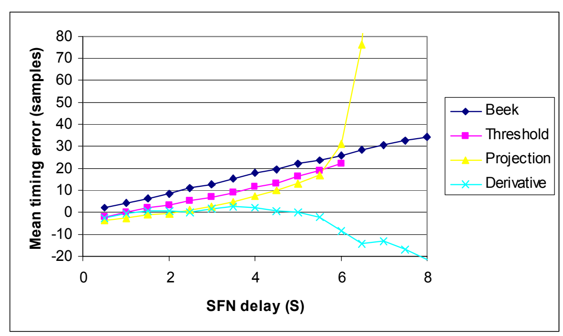


Figure 3. Effect of SFN delay; equal SFN signal powers, Eb/No=20dB

Fig. 4 shows the bit error performance for the Beek and derivative techniques compared to the case with ideal timing. The derivative technique shows a close correspondence to the ideal case across all channel parameters. Investigations for other channels show similar results.

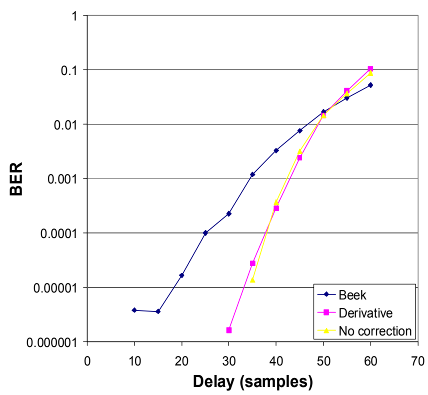


Figure 4. BER performance; 0dB relative SFN power, Eb/No=20dB.

## V. VARIANCE REDUCTION

The processing associated with the derivative method uses a linear fitting procedure which is prone to giving occasional large errors. These were removed to a degree with the combination of a median filter and an averaging FIR filter. While performance is better than existing techniques, it would be beneficial to reduce the variance of the estimates further. This section considers how additional information from the correlation and derivative functions can help to reduce the variance by identifying unlikely estimates, and replacing them with ones more consistent with current information, and past estimates. In particular we consider the position of the peaks of these functions.

Denote each rule with index *i* as $R_i(t_E)$, where $t_E$ as the time index of estimates (incremented per OFDM symbol). The value of the limit itself will be denoted $L_i(t_E)$. For example, based on the behaviour of the correlation and derivative functions, the following rules are proposed:

1. The timing estimate cannot be later than the peak of the correlation function output. Denote this as $R_1(t_E)$.
2. The timing estimate cannot be earlier than the correlation function peak minus the CP length, since the peak will always be within the CP interval. Denote this as $R_2(t_E)$.
3. The timing estimate cannot be earlier than the peak of the derivative function, since the timing point is the breakpoint after the peak. Denote this as $R_3(t_E)$.

Figure 5 shows the block diagram of the process.

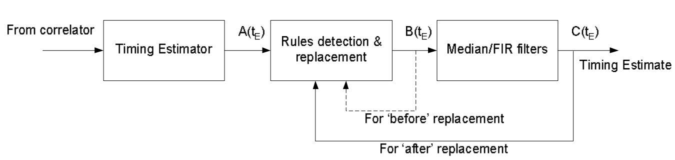


Figure 5. Block diagram of rules processing

Having identified an estimate is likely to be in error, this must be replaced with an estimate that is more consistent with the imposed limits and possibly the previous estimates as well. Three estimation replacement approaches are proposed, and have been investigated. Referring to Figure 5 they are:

1. Hard replacement. When a limit $R_i(t_E)$ is exceeded the estimate is replaced by the limit, i.e. $B(t)= L_i(t_E)$.
2. 'Before' replacement. Replace by previous input to the estimate filter, i.e. $B(t_E)= B(t_E-1)$.
3. 'After' replacement. Replace by previous output of the estimate filter, i.e. $B(t_E)= C(t_E-1)$.

There may be situations where more than one rule is broken and the results may conflict, e.g. $R_1(t_E)$ and $R_3(t_E)$. For this study the rules were tested in the order presented above. For the case when $R_2(t_E)$ and $R_3(t_E)$ are both broken, sequential checking will ensure the earliest estimate is taken since, as previously discussed, it is preferable to advance the estimate than to delay it. For $R_1(t_E)$ and $R_3(t_E)$ both broken there is a conflict since $R_1(t_E)$ wants to delay the estimate, and $R_3(t_E)$ wants to advance it. Again, with this ordering the most advanced option is chosen.

The DVB-T 2k mode simulation as previously described has been used. While previously a median filter of length 15 and an FIR filter of length 16 were used, shorter filters of lengths 5 and 8 respectively have also been used.

The results show a small increase in mean timing error of a few points when the rules are applied, but the mean timing error shows a flatter characteristic. More significant changes are seen in the standard deviation of the estimation error, and for brevity only these are shown.

Figure 6 show the performance as the relative delay between multipath clusters is varied. This shows that replacing bad estimates with the last output from the estimate filter gives a consistent improvement in performance. Also shown is the result for the same algorithm with a shorter estimate filter; it is clear that there is only a degradation of a few samples in the standard deviation. Thus for a small loss of performance much reduced filtering could be employed. Shorter filters will reduce latency through the estimation process and so the estimator can track faster moving channels. The benefits of rules processing is more significant for more complex channel responses.

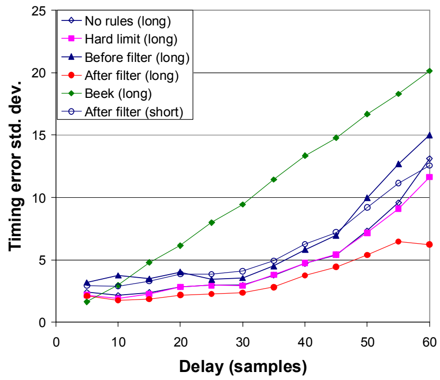


Figure 6. Performance with rules processing, 0dB relative SFN power, Eb/No=20dB..

## VI. DISSCUSSION

A review of existing OFDM synchronisation techniques suggested that they would performance poorly in severe multipath environments, and this has been demonstrated for SFN type channels. A new technique based on the derivative of the summed correlation function has been described and the performance even in the worst case of very short CPs has shown to be superior to the peak detection method. The error variance can be further reduced by exploiting knowledge of the correlation and derivative functions, which has been demonstrated. In considering complexity the synchronisation algorithms are dominated by the correlation calculation, and the additional complexity of the derivative and LS fitting are less than an additional 10%. While this has been presented based on correlation of the cyclic prefix, it can also be applied in repeated symbol preambles, such as WLANs.

## REFERENCES

[1] J-J van de Beek, M. Sandell, P.O. Borjesson, ‘ML Estimation of Time and Frequency Offset in OFDM Systems’, IEEE Trans. Sig. Proc., Vol. 45, No. 7, pp.1800-1805, July 1997.

[2] T.M. Schmidl, D.C. Cox, ‘Robust Frequency and Timing Synchronisation for OFDM’, IEEE Trans. Com., Vol. 45, No. 12, pp.1613-1621, Dec. 1997.

[3] A. Palin, J. Pikkarainen, J. Rinne, ‘Improved symbol synchronization method in OFDM system in channels with large delay spreads’, in Proc. CSDSP'98 (Sheffield, UK), pp. 309-312, April 1998.

[4] EN 300 744, ‘Digital Video Broadcasting (DVB)’, ETSI, 2001.

[5] Bug, S.; Wengerter, C.; Gaspard, I.; Jakoby, R.: "WSSUS-Channel Models for Broadband Mobile Communication Systems", IEEE VTC Spring 2002, Birmingham, Alabama, May 2002.